

# Dynamics study of $Z^+(4430)$ and $X(3872)$ in molecular picture

Xiang Liu<sup>\*</sup>, Yan-Rui Liu<sup>†</sup> and Wei-Zhen Deng<sup>\*\*</sup>

<sup>\*</sup>*Department of Physics, Peking University, Beijing 100871, China  
Centro de Física Teórica, Departamento de Física, Universidade de Coimbra, P-3004-516,  
Coimbra, Portugal*

<sup>†</sup>*Institute of High Energy Physics, P.O. Box 918-4, Beijing 100049, China*

<sup>\*\*</sup>*Department of Physics, Peking University, Beijing 100871, China*

**Abstract.** In this talk, we review our recent work about the dynamical studies of  $Z^+(4430)$  and  $X(3872)$ .  $Z^+(4430)$  can not be explained as a  $D_1' D^*$  or  $D_1 D^*$  molecular state only considering one pion exchange potential without the cutoff, which needs to be confirmed by introducing sigma exchange potential and adding the cutoff in the effective potential. One also excludes the possibility of  $X(3872)$  as a  $DD^*$  molecular state by one pion and one sigma exchanges with the cutoff. Fortunately there exists an S-wave  $BB^*$  bound state with  $J^{PC} = 1^{++}$ . we suggest future experiment to search this state.

**Keywords:** molecular state, potential model, effective chiral Lagrangian

**PACS:** 12.39.Pn, 12.40.Yx, 13.75.Lb

## INTRODUCTION

$Z^+(4430)$ , a new enhancement announced by Belle Collaboration, has stimulated theorists to speculate its underlying structure. Its mass and width are respectively  $m = 4433 \pm 4(\text{stat}) \pm 1(\text{syst})$  MeV and  $\Gamma = 44_{-13}^{+17}(\text{stat})_{-11}^{+30}(\text{syst})$  MeV. The isospin and G-parity of  $Z^+(4430)$  are  $I^G = 1^+$  because  $Z^+(4430)$  was observed in the  $\psi' \pi^+$  channel [1]. The explanations for its structure mainly include the S-wave threshold effect [2], the  $D_1 D^*$  molecular state [3, 4, 5], the tetraquark state [6, 7, 8], the cusp effect [9] and the  $\Lambda_c - \Sigma_c^0$  bound state [10]. Our recent work [12] reviewed the recent theoretical status of  $Z^+(4430)$  [2, 3, 4, 5, 6, 7, 8, 9, 10, 11] and explored whether  $Z^+(4430)$  can be explained as  $D_1' D^*$  or  $D_1 D^*$  molecular state by one pion exchange (OPE).

In a series of XYZ charmonium-like states observed in recent years,  $X(3872)$  [13] is also a state near the threshold of  $DD^*$ , which attracted extensive concerns of theorists [14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Among these theoretical explanations, the molecule picture is the most popular one [14, 15, 16, 17, 18] even though the predictions in  $DD^*$  molecule picture are inconsistent with the experimental measurements to some extent. In fact, only dynamics studies can give a reasonable answer about whether  $X(3872)$  can be interpreted as a  $DD^*$  molecular state. Swanson proposed that  $X(3872)$  was mainly a  $D^0 \bar{D}^{*0}$  molecule bound by both the pion exchange and quark exchange [17]. In Ref. [16], Wong studied the  $DD^*$  system in the quark model in terms of a

---

<sup>1</sup> e-mail: xiangliu@pku.edu.cn, liuxiang@teor.fis.uc.pt

four-body non-relativistic Hamiltonian with pairwise effective interactions, and found an S-wave  $DD^*$  molecule with the binding energy  $\sim 7.53$  MeV. However, with the obtained one pion exchange potential (OPEP) by using the effective Lagrangian, Suzuki argued that  $X(3872)$  is not a molecular state of  $D^0\bar{D}^{*0} + \bar{D}^0D^{*0}$  [24], which contradicts Swanson and Wong's conclusion. In our recent work, we reexamined whether  $X(3872)$  is a molecular state by adding  $\sigma$  meson exchange potential and introducing the cutoff in the effective potential [25].

In this talk, we will briefly introduce the theoretical framework of deducing the effective potential. Then we will respectively discuss whether  $Z^+(4430)$  can be  $D_1' D^*$  or  $D_1 D^*$  bound state and whether  $X(3872)$  can be explained as  $DD^*$  molecular state based on our recent work presented in Ref. [12, 25]. In the last section, a summary will be given.

## THEORETICAL FRAMEWORK

To derive the effective potential, firstly we write out the elastic scattering amplitudes of system according to the Lagrangian, which is constructed in the chiral and heavy quark dual limits [26, 27]

$$\begin{aligned}\mathcal{L} = & ig\text{Tr}[H_b\not{A}_{ba}\gamma_5\bar{H}_a] + ig'\text{Tr}[S_b\not{A}_{ba}\gamma_5\bar{S}_a] + ig''\text{Tr}[T_{\mu b}\not{A}_{ba}\gamma_5\bar{T}_a^\mu] \\ & + [ih\text{Tr}[S_b\not{A}_{ba}\gamma_5\bar{H}_a] + h.c.] + \{i\frac{h_1}{\Lambda_\chi}\text{Tr}[T_b^\mu(D_\mu\not{A})_{ba}\gamma_5\bar{H}_a] + h.c.\} \\ & + \{i\frac{h_2}{\Lambda_\chi}\text{Tr}[T_b^\mu(D\not{A}_\mu)_{ba}\gamma_5\bar{H}_a] + h.c.\} + g_\sigma\text{Tr}[H\sigma\bar{H}],\end{aligned}\quad (1)$$

where  $H_a = \frac{1+\not{v}}{2}[P_a^{*\mu} - P_a\gamma_5]$ ,  $S_a = \frac{1+\not{v}}{2}[P_{1a}^\mu\gamma_\mu\gamma_5 - P_{0a}^*]$  and  $T_a^\mu = \frac{1+\not{v}}{2}\{P_{2a}^{*\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}P_{1a}^\nu\gamma_5[g_\nu^\mu - \frac{1}{3}\gamma_\nu(\gamma^\mu - v^\mu)]\}$ . The axial vector field  $A_{ab}^\mu$  is defined as  $A_{ab}^\mu = \frac{1}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)_{ab}$  with  $\xi = \exp(i\mathcal{M}/f_\pi)$ ,  $f_\pi = 132$  MeV and  $\mathcal{M}$  is the octet pseudoscalar matrix.

We impose the constraint on the scattering amplitudes that initial states and final states should have the same angular momentum. The molecular state  $|J, J_z\rangle$  composed of the  $1^-$  and  $1^+$  charm meson pair can be constructed as

$$|J, J_z\rangle = \sum_{\lambda_1, \lambda_2} \langle 1, \lambda_1; 1, \lambda_2 | J, J_z \rangle |p_1, \varepsilon_1; p_2, \varepsilon_2\rangle \quad (2)$$

where  $\langle 1, \lambda_1; 1, \lambda_2 | J, J_z \rangle$  is the Clebsch-Gordan coefficient. Combining the equation with the scattering amplitudes, one gets the matrix element  $i\mathcal{M}(J, J_z)$ .

With the Breit approximation, the interaction potential in the momentum space is related to  $i\mathcal{M}(J, J_z)$

$$V(q) = -\frac{1}{\sqrt{\prod_i 2m_i \prod_f 2m_f}} \mathcal{M}(J, J_z) \quad (3)$$

where  $m_i$  and  $m_f$  denote the masses of the initial and final states respectively. Then we average the potential in the momentum space. Finally we make Fourier transformation to derive the potential in the coordinate space.

### IS $Z^+(4430)$ A LOOSELY $D'_1 - D^*$ OR $D_1 - D^*$ MOLECULAR STATE?

If  $Z^+(4430)$  is a  $D'_1 D^*$  or  $D_1 D^*$  molecular state, the flavor wave function of  $Z^+(4430)$  is

$$|Z^+\rangle = \frac{1}{\sqrt{2}}(|\bar{D}_1^0 D^{*+}\rangle + |\bar{D}^{*0} D_1^+\rangle), \quad \text{or} \quad |Z^+\rangle = \frac{1}{\sqrt{2}}(|\bar{D}_1^0 D^{*+}\rangle + |\bar{D}^{*0} D_1^+\rangle).$$

For the flavor wave function of  $\tilde{Z}^+$  with opposite G-parity, we only replace the plus sign in the above functions with a minus sign [12].

We only consider the contribution from OPE and obtain the potentials of  $D'_1 D^*$  and  $D_1 D^*$  systems in the coordinate space, which are listed in Table 1.

**TABLE 1.** The one pion exchange potential in the coordinate space with  $A' = \bar{D}_1^0 D^{*+}$ ,  $B' = D_1^+ \bar{D}^{*0}$ ,  $C' = \bar{D}_1^0 D^{*+}$  and  $D' = D_1^+ \bar{D}^{*0}$ . Here  $\zeta = \delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r}$ ,  $\eta = \frac{\cos(\mu r)}{r}$ ,  $\xi = \delta(\mathbf{r}) - \frac{m_\pi^2}{4\pi r} e^{-m_\pi r}$ ,  $\chi = \nabla^2 \delta(\mathbf{r}) - \mu^2 \delta(\mathbf{r}) - \frac{\mu^4}{4\pi} \frac{\cos \mu r}{r}$ .

	$D'_1 - D^*$ system		$D_1 - D^*$ system	
	$A'(B') \rightarrow A'(B')$	$A'(B') \rightarrow B'(A')$	$C'(D') \rightarrow C'(D')$	$C'(D') \rightarrow D'(C')$
$0^-$	$\frac{gg'}{3f_\pi^2} \zeta$	$\frac{h^2(q^0)^2}{8\pi f_\pi^2} \beta$	$-\frac{5gg''}{18f_\pi^2} \xi$	$\frac{h^2}{6f_\pi^2} \chi$
$1^-$	$\frac{gg'}{6f_\pi^2} \zeta$	$\frac{h^2(q^0)^2}{8\pi f_\pi^2} \beta$	$-\frac{5gg''}{36f_\pi^2} \xi$	$-\frac{h^2}{12f_\pi^2} \chi$
$2^-$	$-\frac{gg'}{6f_\pi^2} \zeta$	$\frac{h^2(q^0)^2}{8\pi f_\pi^2} \beta$	$\frac{5gg''}{36f_\pi^2} \xi$	$\frac{h^2}{60f_\pi^2} \chi$

Where  $g = 0.59 \pm 0.07 \pm 0.01$  can be extracted by fitting the experimental width of  $D^*$  [28]. In quark model, Falk and Luke give an approximate relation  $|g'| = |g|/3$  and  $|g''| = |g|$  [26]. With the available experimental information, Casalbuoni and collaborators extracted  $h = -0.56 \pm 0.28$  and  $h' = (h_1 + h_2)/\Lambda_\chi = 0.55 \text{ GeV}^{-1}$  [27]. Besides those coupling constants, other parameters include:  $m_{D^*} = 2007 \text{ MeV}$ ,  $m_{D'_1} = 2430 \text{ MeV}$ ,  $m_{D_1} = 2420 \text{ MeV}$ ,  $m_{B^*} = 5325 \text{ MeV}$ ,  $m_{B'_1} = 5732 \text{ MeV}$ ,  $f_\pi = 132 \text{ MeV}$ ,  $m_\pi = 135 \text{ MeV}$  [29];  $m_{B_1} = 5725 \text{ MeV}$  [30].

With the potentials derived above, we use the variational method to investigate whether there exists a loosely bound state. Our criteria of the formation of a possible loosely bound molecular state is (1) the radial wave function extends to 1 fm or beyond and (2) the minimum energy of the system is negative. Our trial wave functions include (a)  $\psi(r) = (1 + \alpha r)e^{-\beta r}$ ; (b)  $\psi(r) = (1 + \alpha r^2)e^{-\beta r^2}$ ; (c)  $\psi(r) = r^2(1 + \alpha r)e^{-\beta r}$ .

Unfortunately we does not find a solution to satisfy the above criteria for the system of  $D'_1 - D^*$  or  $D_1 - D^*$  in all  $J^P = 0^-, 1^-, 2^-$  channels with the realistic coupling constants

$g = 0.59$ ,  $g' = g/3$  and  $g'' = g$  deduced from the width of  $D^*$ ,  $D_1$  and  $D'_1$ . Such a solution also does not exist if we switch the sign of  $gg'$  or enlarge the absolute value of  $gg'$  by a factor 3. The same conclusion holds for the system of  $B'_1 B^*$ ,  $B_1 B^*$  and  $\tilde{Z}^+$  with negative  $G$ -parity.

It's interesting to note that the one pion exchange potential alone does not bind the deuteron in nuclear physics either. In fact, the strong attractive force in the intermediate range is introduced in order to bind the deuteron, which is sometimes modeled by the sigma meson exchange. One may wonder whether the similar mechanism plays a role in the case of  $Z^+(4430)$  and  $X(3872)$ . Further work along this direction is in progress [31]. Basing on the above considerations, we reanalyze  $DD^*$  system.

### IS $X(3872)$ REALLY A $D - D^*$ MOLECULAR STATE?

We reanalyze the flavor function of  $X(3872)$  and obtain

$$|X(3872)\rangle = \frac{1}{\sqrt{2}} \left[ |D^0 \bar{D}^{*0}\rangle - |D^{*0} \bar{D}^0\rangle \right], \quad (4)$$

which is naturally reflect the positive C-parity of  $X(3872)$ .

With the convention of the  $X(3872)$  flavor wave function in Eq. (4), the potential in the study of the molecular picture finally reads as

$$V(\mathbf{r}) = g_\sigma^2 Y_\sigma(\mathbf{r}) + \frac{g^2}{6f_\pi^2} Y_\pi(\mathbf{r}) \quad (5)$$

with  $Y_\sigma(\mathbf{r}) = \frac{1}{4\pi r} e^{-m_\sigma r}$  and  $Y_\pi(\mathbf{r}) = -\delta(\mathbf{r}) - \frac{\mu^2}{4\pi r} \cos(\mu r)$ , where  $\mu = \sqrt{q_0^2 - m_\pi^2}$ . The sign between one sigma exchange potential (OSEP) and OPEP is determined by the relative sign of  $|D^0 \bar{D}^{*0}\rangle$  and  $|D^{*0} \bar{D}^0\rangle$  in the wave function in Eq. (4).

Due to the existence of the three dimensional  $\delta$  function in the potential, Suzuki argued that  $D$  and  $\bar{D}^*$  could not be bound as a molecular state [24]. We note that the potential in Eq. (5) is derived with the implicit assumption that all the mesons are point-like particles. Such an assumption is not fully reasonable due to the structure effect in every interaction vertex. Thus in the following we will introduce a cutoff to regulate the potential and further study whether it is possible to find a loosely bound molecular state using the realistic potential. We adopt two approaches: (1) considering the form factor (FF) contribution; (2) smearing the potential. Although these two approaches look different, they are essentially the same, i.e. imposing a short-distance cutoff to improve the singularity of the effective potential.

With introducing monopole FF  $F(q) = (\Lambda^2 - m^2)/(\Lambda^2 - q^2)$  in the potential as an example, we give the modified potential as

$$Y_\sigma(r) = \frac{1}{4\pi r} (e^{-m_\sigma r} - e^{-\Lambda r}) - \frac{\eta'^2}{8\pi\Lambda} e^{-\Lambda r}, \quad Y_\pi(r) = -\frac{\mu^2}{4\pi r} [\cos(\mu r) - e^{-\alpha r}] - \frac{\eta^2 \alpha}{8\pi} e^{-\alpha r},$$

where  $\eta = \sqrt{\Lambda^2 - m_\pi^2}$ ,  $\eta' = \sqrt{\Lambda^2 - m_\sigma^2}$  and  $\alpha = \sqrt{\Lambda^2 - q_0^2}$ . Note we use the same  $\Lambda$  for  $\pi$  and  $\sigma$  exchange. We found that the  $\sigma$  exchange potential is repulsive.

One gets numerical solutions depicted in Table 2. We only use the coupling constant  $g_\sigma = 0.76$  to illustrate the results. We chose the solutions with  $-5.0 \text{ MeV} < E_0 < -0.1 \text{ MeV}$ . We found (1)  $DD^*$  interaction through one pion and one sigma exchanges

**TABLE 2.** Solutions for various  $g$  and  $\Lambda$  in the case of FF with total potential. Lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected. Here  $g_\sigma = 0.76$  is used. Here  $r_{\text{rms}}$  is the root-mean-square radius, and  $r_{\text{max}}$  is the radius corresponding to the maximum of the wave function  $\chi(r)$ .

	$\Lambda$ (GeV)	$E_0$ (MeV)	$r_{\text{rms}}$ (fm)	$r_{\text{max}}$ (fm)
$g = 0.59$	6.0	-1.3	2.8	0.1
	6.1	-4.9	1.5	0.1
$g = 0.8$	3.3	-0.7	3.8	0.3
	3.4	-3.7	1.7	0.2
$g = 1.0$	2.1	-0.3	5.9	0.4
	2.2	-2.4	2.2	0.3

**TABLE 3.** Solutions for various  $g$  and  $\Lambda$  in the case of FF for the  $BB^*$  system with the total potential. The lowest eigenvalues between -5.0 MeV and -0.1 MeV are selected. Here  $g_\sigma = 0.76$  is used.

	$\Lambda$ (GeV)	$E_0$ (MeV)	$r_{\text{rms}}$ (fm)	$r_{\text{max}}$ (fm)
$g = 0.59$	2.5	-0.5	2.7	0.3
	2.6	-2.5	1.2	0.2

is not attractive enough to form a bound state with  $g = 0.59$  and  $g_\sigma = 0.76$  and  $\Lambda = 1$  GeV; (2) when  $g$  becomes larger, the critical point for  $\Lambda$  to generate a  $DD^*$  bound state becomes small. The  $BB^*$  system also is investigated. The results are shown in Table 3.

The results for the case of smearing also confirm the above observation for the case of FF.

## SUMMARY

In a short summary, our numerical results indicate that it is hard to explain  $Z^+(4430)$  as a  $D'_1 D^*$  or  $D_1 D^*$  only considering OPEP without cutoff. However this conclusion needs to be confirmed by considering OSEP and adding cutoff in potential, which is in progress. Then a decisive conclusion about whether  $Z^+(4430)$  can be understood as a  $D'_1 D^*$  or  $D_1 D^*$  molecular state can be made.

$X(3872)$  can not be explained as a  $DD^*$  molecular state by considering one pion and one sigma exchanges, and introducing cutoff in the potential. We also find that there exists an S-wave  $BB^*$  system with  $J^P = 1^{++}$ , which can be searched in further experiment.

**Acknowledgments.** We enjoy the collaboration with Professor Shi-Lin Zhu. This project was supported by the National Natural Science Foundation of China under

Grants 10625521, 10675008, 10705001, 10775146, 10721063 and the China Postdoctoral Science foundation (20060400376, 20070420526). X.L. specially thanks the support of the *Fundação para a Ciência e a Tecnologia of the Ministério da Ciência, Tecnologia e Ensino Superior* of Portugal (SFRH/BPD/34819/2007).

## REFERENCES

1. Belle Collaboration, K. Abe et al., arXiv:0708.1790 [hep-ex].
2. J.L. Rosner, Phys. Rev. **D 76**, 114002 (2007).
3. C. Meng and K.T. Chao, arXiv:0708.4222 [hep-ph].
4. G.J. Ding, arXiv:0711.1485 [hep-ph].
5. S.H. Lee, A. Mihara, F.S. Navarra and M. Nielsen, arXiv:0710.1029 [hep-ph].
6. L. Maïania, A.D. Polosab and V. Riquerb, arXiv:0708.3997 [hep-ph].
7. K. Cheunga, W.Y. Keung and T.C. Yuan, Phys. Rev. **D 76**, 117501 (2007).
8. S.S. Gershtein, A.K. Likhoded and G.P. Pronko, arXiv:0709.2058 [hep-ph].
9. D.V. Bugg, arXiv:0709.1254 [hep-ph]; arXiv:0802.0934 [hep-ph].
10. C.F. Qiao, arXiv:0709.4066 [hep-ph].
11. E. Braaten and M. Lu, arXiv:0712.3885 [hep-ph].
12. X. Liu, Y.R. Liu, W.Z. Deng and S.L. Zhu, Phys. Rev. **D 77**, 034003 (2008), arXiv:0711.0494.
13. Belle Collaboration, S.K. Coi et al., Phys. Rev. Lett. **91**, 262001 (2003); CDF Collaboration, D. Acosta et al., Phys. Rev. Lett. **93**, 072001 (2004); D0 Collaboration, V.M. Abazov et al., Phys. Rev. Lett. **93**, 162002 (2003); Babar Collaboration, B. Aubert et al., Phys. Rev. **D 71**, 071103 (2005); Belle Collaboration, K. Abe et al., arXiv: hep-ex/0505037; Babar Collaboration, B. Aubert, Phys. Rev. **D 74**, 071101 (2006); Belle Collaboration, G. Gokhroo et al., Phys. Rev. Lett. **97**, 162002 (2006); Babar Collaboration, talk given by P. Grenier in Moriond QCD 2007, 17-24 March, <http://moriond.in2p3.fr/QCD/2007/SundayAfternoon/Grenier.pdf>; CDF Collaboration, A. Abulencia et al., Phys. Rev. Lett. **96**, 102002 (2006).
14. F.E. Close, P.R. Page, Phys. Lett. **B 578**, 119 (2004).
15. M.B. Voloshin, Phys. Lett. **B 579**, 316 (2004).
16. C.Y. Wong, Phys. Rev. **C 69**, 055202 (2004).
17. E.S. Swanson, Phys. Lett. **B 588**, 189 (2004); *ibid* B 598, 197 (2004).
18. N.A. Tornqvist, Phys. Lett. **B 590**, 209 (2004).
19. D.V. Bugg, Phys. Lett. **B 598**, 8 (2004).
20. J.L. Rosner, Phys. Rev. **D 74**, 076006 (2006).
21. B.A. Li, Phys. Lett. **B 605**, 306 (2005).
22. L. Maïani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. **D 71**, 014028 (2005).
23. H. Hogaasen, J.M. Richard, P. Sorba, Phys. Rev. **D 73**, 054013 (2006); D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Lett. **B 634**, 214 (2006); N. Barnea, J. Vijande, A. Valcarce, Phys. Rev. **D 73**, 054004 (2006); Y. Cui, X.L. Chen, W.Z. Deng, S.L. Zhu, High Energy Phys. Nucl. Phys. **31**, 7 (2007), arXiv: hep-ph/0607226; R.D. Matheus, S. Narison, M. Nielsen, J.M. Richard, Phys. Rev. **D 75**, 014005 (2007); T.W. Chiu et al., Phys. Lett. **B 646**, 95 (2007); Phys. Rev. **D 73**, 111503 (2006), Erratum-*ibid.* **D 75**, 019902 (2007); K. Terasaki, arXiv:hep-ph/0706.3944 [hep-ph].
24. M. Suzuki, Phys. Rev. **D 72**, 114013 (2005).
25. Y.R. Liu, X. Liu, W.Z. Deng and S.L. Zhu, arXiv:0801.3540.
26. A. F. Falk and M. Luke, Phys. Lett. **B 292**, 119 (1992).
27. R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto and G. Nardulli, Phys. Rept. **281**, 145 (1997).
28. CLEO Collaboration, S. Ahmed et al., Phys. Rev. Lett. **87**, 251801 (2001).
29. W. M. Yao et al., Particle Data Group, J. Phys. G **33**, 1 (2006).
30. CDF Collaboration,  $\alpha$ Mass and width measurement of orbitally excited ( $L = 1$ )  $B^{*0}$  mesons, CDF Note 8945, [www-cdf.fnal.gov/physics/new/bottom/070726.blessed-bss/](http://www-cdf.fnal.gov/physics/new/bottom/070726.blessed-bss/); CDF Collaboration, Andreas Gessler, the EPS talk given on HEP 2007 in Manchester, arXiv:0709.3148 [hep-ex].
31. X. Liu, Y.R. Liu, W.Z. Deng and S.L. Zhu, arXiv:0803.1295 [hep-ph].